Exercise 1

1. Using DeMorgan's rule, state the negation of the statement: "The car is out of gas or the fuel line is plugged."

   (a) The car has gas or the fuel line is unplugged

   (b) The car has gas and the fuel line is unplugged

   (c) The car is out of gas and the fuel line is plugged

   (d) The car is out of gas or the fuel line is plugged

   Ans : b. A=The car is out of gas, B=the fuel line is plugged . Then negation of $A \cup B = (A \cup B)^c = A^c \cap B^c$

2. How many functions are there from $\{0,1\}^n$ to $\{1,2,\ldots,n\}$.

   (a) n^{2^n}

   (b) $n2^n$

   (c) 2^n

   (d) $n + 2^n$

   Ans : a. # of functions from a set A to B is precisely $|B|^{|A|}$. In this case $|B| = n$ and $|A| = 2^n$.

3. Let $D = \{-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36\}$. Determine which of the following statements are true and which are false. Provide counterexamples for those statements that are false. Prove the statements that are true.

   (a) $\forall x \in D$, if x is odd then $x > 0$.

   (b) $\forall x \in D$, if x is less than 0 then x is even.

   (c) $\forall x \in D$, if x is even then $x \leq 0$.

   (d) $\forall x \in D$, if the ones digit of x is 6, then the tens digit is 1 or 2.

   Ans : a & b. Trivial to check.

4. If A and B are two sets such that $|A| = 8$ and $|B| = 9$ and $|A \cup B| = 15$ then what is $|A \cap B|$?

   (a) 0

   (b) 1

   (c) 4

   (d) 2

   Ans : d. $|A \cap B| = |A| + |B| - |A \cup B| = 8 + 9 - 15 = 2$

5. A pair of numbers x and y satisfy a system of inequalities if

$$\begin{cases} 3 \leq x \leq 5 \text{ and} \\ |x - y| < 1. \end{cases}$$

   What are the conditions under which x and y fail to satisfy this system?

   (a) $x < 3$ or $x > 5$ or $|x - y| < 1$

   (b) $3 \leq x \leq 5$ or $|x - y| < 1$

   (c) $x < 3$ or $x > 5$ or $x \geq y + 1$ or $y \geq x + 1$

   (d) $x \leq 3$ or $x \geq 5$ or $x > y + 1$ or $y > x + 1$

   Ans : c. It fails when one of the conditions $3 \leq x \leq 5$ or $|x-y| < 1$ is violated. First condition violated means $x < 3$ or $x > 5$. Second condition implies either $x < y + 1$ or $x > y - 1$. That is violated when $x \geq y + 1$ or $y \geq x + 1$.

6. If $S = \{0, 1\}$ and $T = \{a, e, i, o, u\}$ then what is the number of relations between S and T ?

   Ans : Number of relations is explicitly $2^{|S||T|}$. In this case $|S| = 2, |T| = 5$. Hence # relations $= 2^{10} = 1024$.

7. If A, B are two sets such that $|A| = 3$ and $|B| = 5$, then what are the possible values of $A \cup B$? Multiple answers can be correct.

   (a) 3

   (b) 5

   (c) 8

   (d) 9

   Ans : b & c . a can not be correct because $|A \cup B| \geq 5$.(All elements of A and B are in the union . As B has at least 5 elements those elements are there in the union . Hence the size is at least 5) . It can not be bigger than 8 because even if all the elements of A and B are different then maximum 8 elements can be there in the union. Hence d is not possible. 5 is indeed possible when $A \subset B$. c possible when $A \cap B = \phi$.

8. If a, b, c, d are positive integers such that a divides c and b divides d, then is it true that $a+b$ divides $c+d$?

   (a) True

   (b) False

   Ans : b . Example take $a = 2, c = 4, b = 5, d = 15$. 2 divides 4 , 5 divides 15 but $2 + 5 = 7$ does not divide $4 + 15 = 19$.

9. If $S = \{0, 1\}$ and $T = \{a, e, i, o, u\}$ then what is $|S \times T|$?

   Ans : Precisely $|S| \times |T| = 2 \times 5 = 10$.